ELEMENTS OF PHYSICS
NINTH EDITION

Alpheus W. Smith
Late Professor of Physics, Ohio State University

John N. Cooper
Professor of Physics, U.S. Naval Postgraduate School

McGRAW-HILL BOOK COMPANY
New York  St. Louis  San Francisco  Auckland  Bogotá  Düsseldorf
Johannesburg  London  Madrid  Mexico  Montreal  New Delhi
Panama  Paris  São Paulo  Singapore  Sydney  Tokyo  Toronto
When this value is combined with Millikan’s much later determination of \( e = 1.602 \times 10^{-19} \) C, we find that the mass \( m \) of the electron is \( 9.11 \times 10^{-31} \) kg.

In Chap. 35 we found that it requires \( 9.649 \times 10^7 \) C to deposit electrolytically 1 kg equivalent mass of any element. In the case of hydrogen, this corresponds to 1.008 kg. If the cathode rays in a discharge tube bear the same magnitude of charge as a monovalent ion, we can calculate the ratio of the mass \( M \) of a hydrogen atom to that of the electron. For the hydrogen atom \( e/M = 9.649 \times 10^7 \) C/1.008 kg = \( 9.575 \times 10^7 \) C/kg, from which we have

\[
\frac{\text{Mass of hydrogen atom}}{\text{Mass of electron}} = \frac{M}{m} = \frac{1.759 \times 10^{11}}{9.575 \times 10^7} = 1.837
\]

### 41.6 PHOTOELECTRICITY AND THE PHOTON

In 1887, a decade before J. J. Thomson measured \( e/m \) for the electron, Heinrich Hertz was doing pioneer work in the study of radio waves. In the course of his experiments Hertz observed that when ultraviolet light fell on the electrodes of a spark gap, a spark jumped at a substantially lower potential difference than usual. A little later it was found that a charged sheet of zinc exposed to ultraviolet radiation loses its charge when negative but retains its charge when positive (Fig. 41.6). We now know that when ultraviolet light falls on a metallic surface, electrons are liberated from the metal. The energy required to eject an electron is provided by the incident electromagnetic radiation. When the metal is negatively charged, these photoelectrons are repelled. When the metal is positive, the photoelectrons are attracted back to the metal. No quantitative understanding of the photoelectric effect became available until 1905. It required the synthesis of the concept of the electron and the concept of a photon (or quantum), which we now develop.

Near the end of the nineteenth century Max Planck was endeavoring to develop an adequate theory for the wavelength distribution of blackbody radiation (Fig. 16.6). He discovered that he could account quantitatively for blackbody radiation curves if he assumed that energy is radiated in small packets, which he called quanta and for which C. N. Lewis later introduced the name photons. According to Planck’s theory, the energy associated with each quantum or photon is given by

\[
\text{Energy of photon } = W = h\nu
\]  

(41.3)

where \( \nu \) is the frequency of the radiation and \( h \) is a universal constant equal to \( 6.626 \times 10^{-34} \) J·s. In honor of its discoverer, \( h \) is called Planck’s constant.

---

**FIGURE 41.6**

When ultraviolet light falls on a zinc plate, photoelectrons are ejected and the electroscope discharges. The glass plate absorbs the ultraviolet, and when it is inserted in the beam, the photo-emission ceases.

![Diagram of photoelectric effect](image-url)
It was in 1901 that Planck proposed his startling new idea, which now plays a vital role in our understanding of physics in atomic dimensions. Although Planck's theory gave excellent agreement with experimental measurements on blackbody radiation, many people regarded the agreement as fortuitous and considered the proposal that radiant energy comes in chunks or quanta as unacceptable. The objections were based on the misconception that light and other electromagnetic radiation were purely classical wave phenomena. In the early 1800s it had been established that light exhibits interference, diffraction, and polarization (Chaps. 27 and 28), which are characteristic wave properties. Virtually a century of experimentation had established that radiation does indeed have wave characteristics. In the 1860s Maxwell had developed the theory of electromagnetic fields and proposed that light consists of electromagnetic waves, a hypothesis which was strongly supported in 1887 by the experimental work of Hertz. Thus it is not surprising that many physicists regarded Planck's proposal that radiation had particle properties as erroneous and retrograde. According to Einstein's extension of Planck's idea, a quantum is emitted by one atom (or oscillator) and absorbed by another as though it were a particle with a well-defined position. On the other hand, a wave emitted from a source spreads out, and by the time it is a centimeter from the source, it is spread over a volume millions of times that of a single atom.

Now we believe that radiation is neither a classical wave phenomenon nor a classical particle phenomenon. It has wave characteristics under some conditions and particle properties under others. Classical waves and particles are idealizations which are extremely useful, but electromagnetic radiation is neither a pure wave motion nor exclusively particlelike. Instead it exhibits some aspects of each, as we shall see.

41.7 EINSTEIN'S PHOTOELECTRIC EQUATION

A theory like Planck's, which utilizes some new assumption to explain a phenomenon, is called an ad hoc theory. Such theories are usually viewed with skepticism until the radical assumption is found to have wider application. The first of much supporting evidence for the validity of Planck's hypothesis came in 1905, when Einstein used it to explain the photoelectric effect, a phenomenon which had been a puzzle to physicists since its discovery. By 1905 it had been established that electrons are emitted from all metals (and many other materials) when ultraviolet light is incident (but not visible light). On the other hand, there are several elements from which visible light ejects photoelectrons. To explain these facts, Einstein adopted the hypothesis that radiant energy comes in quanta of energy \( h\nu \). In order to eject a photoelectron from a surface, a photon must have enough energy to free the electron from the atom or material with which it is associated. If it requires an energy \( W \) to remove an electron from the metal, and if the photon imparts to it an energy \( h\nu \) which is greater than \( W \), the difference in energy appears as kinetic energy of the ejected photoelectron. This explanation of the photoelectric effect in terms of photons was one of the early triumphs of the quantum idea. If we let \( W_{\text{max}} \) represent the smallest energy which can free an electron from the solid, the maximum kinetic energy of ejected electrons is given by Einstein's photoelectric relation:
By 1914 it was established that x-rays have wave properties and that they are a form of electromagnetic radiation. Well before this, radioactivity had been discovered, and some radioactive materials were found to emit gamma rays, which are of the same fundamental nature as x-rays. Indeed, short-wavelength electromagnetic radiation is called gamma radiation when it comes from a radioactive nucleus and x-radiation when it arises from bombarding a target with high-energy electrons. Since the properties of the rays depend on their wavelengths and not on their origin, we shall discuss radioactivity briefly before we treat the detection and absorption of x-rays and gamma rays in Chap. 44.

Electromagnetic radiation has not only wave characteristics but also particle properties, as pointed out in Sec. 41.6. It comes in photons with energy $h\nu$ [Eq. (41.3)], where $h$ is Planck's constant and $\nu$ is the frequency. Since $\nu = c/\lambda$, the shorter the wavelength the greater the energy associated with a photon. For x-ray and gamma-ray photons the energy ranges from a few hundred to many millions of electronvolts. Further evidences of the particle properties of electromagnetic radiation are presented in the following three sections.

### 43.5 THE CONTINUOUS X-RAY SPECTRUM

Two phenomena from the field of x-rays support the blackbody and the photoelectric evidence for the photon hypothesis. They are the short-wavelength limit of the continuous x-ray spectrum and the Compton effect, which are discussed respectively in this and the following section. As the work of Bragg (Sec. 43.4) and others has shown, the wavelengths of the x-rays emitted when a beam of high-energy electrons strike a target vary over a broad range. There is always a continuous spectrum upon which certain lines characteristic of the target element can be superimposed. When the intensity of the continuous radiation in a given wavelength interval is plotted as a function of wavelength for different values of the applied potential across the tube, curves similar to those of Fig. 43.10 are produced. Three features of these curves are immediately apparent:

1. The intensity radiated increases at all wavelengths when the potential difference across the tube is raised.

2. The shortest wavelength emitted at a given potential is sharply defined and decreases as the voltage across the tube increases.

3. As the potential difference across the tube is increased, the wavelength at which the maximum energy is radiated shifts toward shorter wavelengths.

We can understand why the spectrum is continuous and predict quantitatively the short-wavelength limit if we invoke the Planck hypothesis that electromagnetic radiation is emitted in photons of energy $h\nu$. X-rays are produced when the kinetic energy of the incident electrons is transformed into electromagnetic radiation through collisions with atoms of the target. Most of the collisions are glancing ones, in which only some moderate fraction of an electron's energy is radiated as a photon. Before being stopped, most electrons have several collisions and produce several photons of widely varying wavelengths. Occasionally an electron has a head-on collision, in which it loses all its energy at once. Such collisions
are relatively rare and are the ones which produce the x-rays at the short-wavelength limit. Clearly, the most energy an electron can lose is all it has. This corresponds to $V_e$, where $V$ is the potential difference across the tube and $e$ the electronic charge. If we equate this energy to that of the most energetic photon, we obtain

$$V_e = h
\nu_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} \tag{43.3}$$

where $c$ is the speed of light. Solving for $\lambda_{\text{min}}$ yields

$$\lambda_{\text{min}} = \frac{hc}{V_e} \tag{43.3a}$$

This relationship, known as the Duane-Hunt law (1915), was used for one of the earliest reliable determinations of Planck’s constant $h$.

In many respects continuous x-ray emission is the inverse of the photoelectric effect. In the production of an x-ray photon the kinetic energy of the incident electron is converted into radiant energy; in the photoelectric effect the radiant energy of a photon is converted, at least in part, into kinetic energy of the electron.

The amount of energy available from each electron is increased as the potential difference across the tube is increased, and we should expect more energy to be radiated per electron. Thus the ordinates in Fig. 43.10 increase as the potential difference across the tube is increased. The area under the curve is proportional to the amount of energy radiated in the form of x-rays. The rest of the energy of the incident electrons appears as heat in the target.

### 43.6 SCATTERING AND THE COMPTON EFFECT

When a beam of x-rays passes through matter, some energy is scattered out of the beam. According to the classical theory of J. J. Thomson, the atomic electrons are driven to perform simple harmonic motion by the electric intensity $E$ of the incident waves. Since these electrons are accelerated, they radiate a frequency equal to that of the incident waves. Thus the electrons remove energy from the passing wave and re-radiate this energy in other directions. Scattering of waves with the frequency of the incident radiation is observed over the entire electromagnetic spectrum; however, it is weak at high frequencies. This unmodified scattering can be treated satisfactorily in terms of classical waves.

In 1923 Compton published the results of careful measurements of the x-ray frequencies scattered by carbon atoms upon which monochromatic x-rays were incident (Fig. 43.11). He found that the scattered beam contains two frequencies, one the same as that of the incident beam, and the second somewhat lower. Figure 43.11 shows the wavelength distribution of the radiation scattered at various angles. At each angle radiation is scattered not only at the wavelength of the incident beam but also at a longer wavelength whose value depends on the scattering angle $\theta$ (the angle between the propagation directions of the incident and scattered radiation).

Compton’s measurements showed that the change in wavelength $\Delta \lambda$ is independent of the scattering material but depends on the scattering angle $\theta$ according to the relation
The scattering of Mo Kα x-rays \((\lambda = 0.707 \text{ Å})\) at an angle \(\theta\) produces two peaks in the scattered radiation, one at the wavelength of the incident radiation and the second at a wavelength greater by \(\Delta \lambda = 0.024(1 - \cos \theta) \text{ Å}\).

\[
\Delta \lambda = 0.024(1 - \cos \theta) \text{ Å}
\]  \hspace{1cm} (43.4)

Compton explained the wavelength shift in the scattering of x-rays by assuming that the incident beam of x-rays consists of a stream of photons of energy \(h\nu_0\). These photons possess momentum \(h\nu_0/c\) as well as energy. Their collisions with electrons (Fig. 43.12) can be described in terms of the laws of conservation of momentum and conservation of energy. Applying these laws to a collision, we obtain, for the relativistic case in which the electron is initially at rest,

\[\text{Incident photon} \quad h\nu_0 \quad \text{Energy} \quad h\nu_0 \quad \text{Momentum} \]

\[\text{Scattered photon} \quad h\nu_0/c \quad \text{Energy} \quad h\nu_0/c \quad \text{Momentum} \]

\[\text{Momentum} \quad (p - p_0) \quad \text{of recoil electron} \quad \text{with kinetic energy} \quad (m - m_0)c^2\]
Conservation of energy:
\[ h\nu_0 = h\nu + (m - m_0)c^2 \]  \hspace{1cm} (43.5)

Conservation of momentum:
\[ \begin{align*}
\text{x component:} & \quad \frac{h\nu}{c} = \frac{h\nu}{c} \cos \theta + \frac{m_0\nu}{\sqrt{1 - v^2/c^2}} \cos \phi \\
\text{y component:} & \quad 0 = \frac{h\nu}{c} \sin \theta - \frac{m_0\nu}{\sqrt{1 - v^2/c^2}} \sin \phi
\end{align*} \]  \hspace{1cm} (43.6)\hspace{1cm} (43.7)

where \( \nu \) is the frequency of the scattered photon, \( m \) and \( \nu \) are the mass and speed of the recoil electron, \( m_0 \) is its rest mass, \( \theta \) is the angle through which the photon is scattered, and \( \phi \) is the angle between \( \nu \) and the direction in which the photon was incident. With considerable effort these equations, together with Eq. (42.11), can be solved to find the change in wavelength \( \Delta\lambda = c/\nu - c/\nu_0 \). The result is that
\[ \Delta\lambda = \frac{h}{m_0c}(1 - \cos \theta) = 0.02426(1 - \cos \theta) \ \text{Å} \]  \hspace{1cm} (43.4a)

Compton's experiments were a clear indication that electromagnetic radiation has particle as well as wave properties. The Compton effect is convincing evidence that radiation in the x-ray region comes in photons, or quanta, which behave like particles in collisions with electrons.

### 43.7 PAIR PRODUCTION AND THE POSITRON

Still another phenomenon which requires the photon concept and relativity for explanation was discovered by Anderson in 1932. When photons with energy greater than 1.02 MeV pass through matter, the photon may interact with an atomic nucleus and be transformed into two particles, an electron and a positron. A positron is a particle identical with an electron except that it bears a positive charge. The higher the energy of a photon, the more probable pair production becomes. It requires a minimum energy of 1.02 MeV to provide the rest mass of the electron and the positron. The positron and electron share whatever energy is left over from the energy of the incident photon. Thus, if pairs are produced by a 4.02-MeV photon, the electron and positron share 3.0 MeV of kinetic energy.

Positrons exist for a very short time. As they move through matter, they lose energy rapidly. As soon as they are stopped, an electron and a positron interact to annihilate each other. Two electron masses vanish, and 1.02 MeV of energy is released. This usually appears in the form of two 0.51-MeV photons moving in opposite directions. The two photons are formed rather than a single 1.02-MeV photon because it is not possible to conserve both energy and momentum for a single photon (unless some additional particle is involved). Both in pair production and in pair annihilation the particle properties of photons are in evidence.
with water to release hydrogen. Physically they are all metals with shiny luster and high electric and thermal conductivities. Immediately preceding these alkali metals in the periodic table are the noble gases, helium, neon, argon, krypton, xenon, and radon, all of which are relatively inert chemically. Except for helium, all have outer electron configurations with eight electrons in their outer shell corresponding to the filling of all levels of this shell with \( l = 0 \) and \( l = 1 \). The elements which directly precede the inert gases, fluorine, chlorine, bromine, iodine, and astatine, are called the halogens; all are nonmetals, and all are strongly electronegative, forming salts when they combine with the alkali metals. In general, elements which have similar electron configurations in their outer shells have similar chemical and physical properties.

One physical characteristic of the elements which can be correlated with the way in which electrons are added to fill the various shells is the ionization potential, or the energy required to remove one of the least tightly bound electrons from an atom, shown graphically in Fig. 46.2. Note that for potassium, sodium, lithium, etc., the ionization potential is exceptionally low compared with that of neighboring atoms. This arises from the fact that in each case there is a single electron in a new outer shell. On the other hand, if there are eight electrons in any given outer shell, a peculiarly stable electronic configuration occurs in which the element is chemically inert, the case for the noble gases. Many other properties show a similar periodicity, among them atomic diameter, compressibility, melting point, and coefficient of thermal expansion. A particularly important example of this periodicity is found in the visible spectra of the elements, which arise for any element when one of the outer electrons of an atom is excited from its normal state to an excited one of higher energy. Atoms with similar electron configurations exhibit similar spectra.

46.6 LOW-LYING ATOMIC ENERGY LEVELS

In the preceding section we have seen how the Pauli exclusion principle can be used to give us insight into the periodic table and the chemical and physical properties associated with the outer-electron configuration of an atom. But can we find any evidence that the four quantum numbers \( n, l, j, \) and \( m_l \) are useful for describing the inner structure of atoms? Actually the characteristic x-ray spectra (briefly treated in Sec. 45.5) are emitted when an inner electron makes a transition to a lower energy state made vacant by knocking an electron out of this state. A detailed explanation of the many strong characteristic x-ray lines can be
made by appropriate use of the quantum numbers \( n \), \( l \), and \( j \) on which the energy depends; in the absence of an external field the energy does not depend on \( m_\ell \). Simpler and more direct evidence comes from the photoelectric absorption of x-rays.

With the aid of an x-ray spectrometer and the continuous spectrum from an x-ray tube, a monochromatic beam of x-ray photons can be produced. If a thin foil of some element is placed in the beam, the intensity is reduced from \( I_0 \) to \( I \). If \( t \) is the thickness of the foil, the absorption coefficient is given by \( \frac{\ln (I_0/I)}{t} \). Figure 46.3 is a plot of absorption coefficient for a lead foil as a function of wavelength. At 0.14 Å there is a sharp drop, called the K absorption edge, due to the fact that photons of wavelength greater than 0.14 Å do not have enough energy to eject one of the K electrons. A K electron is characterized by the quantum numbers \( n = 1, l = 0, j = \frac{1}{2} \).

As the wavelength is increased, the absorption increases, until at 0.78 Å the photons no longer have enough energy to eject electrons with \( n = 2, l = 0, j = \frac{1}{2} \), known in x-ray terminology as L\(_1\) electrons. At 0.81 Å comes the L\(_{\text{II}}\) absorption discontinuity associated with two electrons having \( n = 2, l = 1, j = \frac{1}{2}, \frac{3}{2} \) (\( m_\ell = \pm \frac{1}{2} \)) while the L\(_{\text{III}}\) edge at 0.95 Å is due to four electrons with \( n = 2, l = 1, j = \frac{3}{2} \) (\( m_\ell = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2} \)). At longer wavelengths the absorption increases steadily until one comes to five edges between 3 and 5 Å associated with the \( n = 3 \) (M) electrons. Thus in a sufficiently heavy atom one can find an absorption discontinuity associated with each possible value of \( n, l, \) and \( j \) and no others. Table 46.1 lists the energy required to eject an electron from each of the states characterized by all possible values of \( l \) and \( j \) for \( n = 1, 2, \) and 3.

Further confirmation of these low-lying energy states comes from

![Figure 46.3](image_url)

**FIGURE 46.3**
The photoelectric absorption coefficient of lead as a function of x-ray wavelength.
<table>
<thead>
<tr>
<th>Level</th>
<th>$n$</th>
<th>$l$</th>
<th>$j$</th>
<th>X-ray</th>
<th>Al ((Z = 13))</th>
<th>Cu ((Z = 29))</th>
<th>Mo ((Z = 42))</th>
<th>Ag ((Z = 47))</th>
<th>W ((Z = 74))</th>
<th>Pb ((Z = 82))</th>
<th>U ((Z = 92))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0</td>
<td>1</td>
<td>0</td>
<td>1/2</td>
<td>K</td>
<td>1.562</td>
<td>8.996</td>
<td>20.036</td>
<td>25.556</td>
<td>69.637</td>
<td>88.163</td>
<td>115.06</td>
</tr>
<tr>
<td>2 0</td>
<td>2</td>
<td>0</td>
<td>1/2</td>
<td>L$_1$</td>
<td>0.1154</td>
<td>1.104</td>
<td>2.872</td>
<td>3.811</td>
<td>12.115</td>
<td>15.892</td>
<td>21.765</td>
</tr>
<tr>
<td>2 1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>L$_{II}$</td>
<td>0.00730</td>
<td>0.955</td>
<td>2.633</td>
<td>3.529</td>
<td>11.559</td>
<td>15.231</td>
<td>20.974</td>
</tr>
<tr>
<td>2 1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>L$_{III}$</td>
<td>0.00727</td>
<td>0.935</td>
<td>2.528</td>
<td>3.356</td>
<td>10.219</td>
<td>13.061</td>
<td>17.193</td>
</tr>
<tr>
<td>3 0</td>
<td>3</td>
<td>0</td>
<td>1/2</td>
<td>M$_1$</td>
<td>0.122</td>
<td>0.309</td>
<td>0.718</td>
<td>2.821</td>
<td>3.860</td>
<td>5.356</td>
<td>7.176</td>
</tr>
<tr>
<td>3 1</td>
<td>3</td>
<td>1</td>
<td>1/2</td>
<td>M$_2$</td>
<td>0.078</td>
<td>0.413</td>
<td>0.603</td>
<td>2.574</td>
<td>3.566</td>
<td>5.187</td>
<td>6.805</td>
</tr>
<tr>
<td>3 1</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>M$_{II}$</td>
<td>0.076</td>
<td>0.396</td>
<td>0.572</td>
<td>2.279</td>
<td>3.076</td>
<td>4.306</td>
<td>5.310</td>
</tr>
<tr>
<td>3 2</td>
<td>3</td>
<td>2</td>
<td>1/2</td>
<td>M$_{IV}$</td>
<td>0.0003</td>
<td>0.234</td>
<td>0.373</td>
<td>1.879</td>
<td>2.591</td>
<td>3.725</td>
<td>4.306</td>
</tr>
<tr>
<td>3 2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>M$_V$</td>
<td></td>
<td>0.231</td>
<td>0.367</td>
<td>1.807</td>
<td>2.482</td>
<td>3.356</td>
<td></td>
</tr>
</tbody>
</table>

1 Å = angstrom unit is equal to 12.398 (E in keV)

measuring the kinetic energies of photoelectrons ejected from a thin film of some element. If the film is bombarded with photons of energy $h\nu$, the kinetic energy (KE) of a photoelectron ejected from the K shell is given by

$$KE = h\nu - W_K \tag{46.4}$$

where $W_K$ is the binding energy of the electron in the K shell. If the electron comes from the L$_{III}$ shell, its kinetic energy is $h\nu - W_{L_{III}}$ and so forth. Thus, a knowledge of the energy $h\nu$ of the incident photon and the kinetic energy of an ejected photoelectron permits the determination of the energy required to remove the electron from its bound state.

**Example** X-ray photons of 20,000 keV energy fall on a lead foil and eject photoelectrons, one group of which has a kinetic energy of 6,939 keV. (a) What is the binding energy associated with the level from which these electrons are ejected? (b) What kinetic energy would a photoelectron from the L$_1$ level have?

(a) $KE = h\nu - W$

$$6,939 \text{ keV} = 20,000 \text{ keV} - W$$

$$W = 13,061 \text{ keV}$$

which corresponds to the L$_{III}$ level.

(b) $KE = h\nu - W_{L_{II}}$ (see Table 46.1)

$$KE = 20,000 \text{ keV} - 15,892 \text{ keV}$$

$$= 4,108 \text{ keV}$$

The characteristic x-ray lines of an element arise from electron transitions from one of the lower levels to a vacancy in a level still lower. From the energy of these photons and the known energy levels of the atom one can determine the initial and final states of the electron making the transition. Data from absorption edges, the kinetic energies of photoelectrons, and the wavelengths of characteristic spectral lines are all mutually compatible with each other and with the requirements of the Pauli exclusion principle.